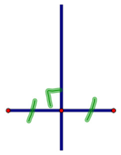
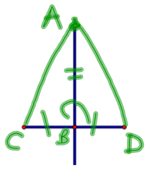
5.2 Use Perpendicular Bisectors
5.3 Use Angle Bisectors of Triangles

perpendicular bisector—is a line, segment, plane, or ray that passes through the midpoint of a segment and is perpendicular to it.



Thm 5.2—Perpendicular Bisector Theorem—In a plane, any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

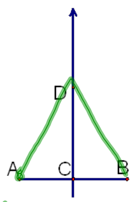


Given: $\overline{AB}$ perp. bis. of $\overline{CD}$
Prove: $\overline{AC}$ $\overline{AD}$

① ~~~~	① Given
② $\overline{CB} \cong \overline{BD}$	② def ⊥ Bisector
③ $\overline{AB} \cong \overline{AB}$	③ Refl.
④ $\angle ABC + \angle ABD$ are rt ∠s	④ def ⊥
⑤ $\angle ABC \cong \angle ABD$	⑤ Rt ∠s ≅
⑥ $\triangle ABC \cong \triangle ABD$	⑥ SAS
⑦ $\overline{AC} \cong \overline{AD}$	⑦ CPCTC

Thm 5.3—Converse of the Perpendicular Bisector Theorem—In a plane, any point equidistant from the endpoints of the segment lies on the perpendicular bisector of the segment.

Examples:

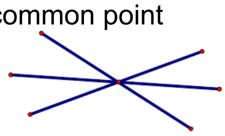


1. $\overline{CD}$ is a ⊥ bisector of $\overline{AB}$.
   m∠DCA = 2x. Solve for x. 45
   $2x = 90$

   AC = 3y + 2, BC = 14. Solve for y. 4
   $3y + 2 = 14$

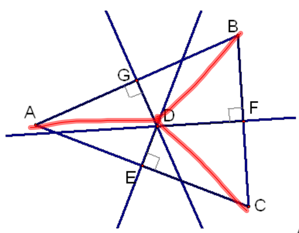
   AD = 4z, BD = 20. Solve for z. 5
   $4z = 20$

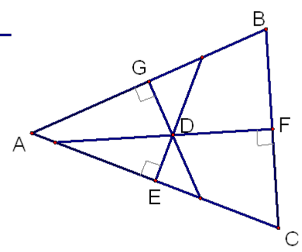
Concurrent lines—three or more lines that intersect at a common point



Point of concurrency—the point of intersection

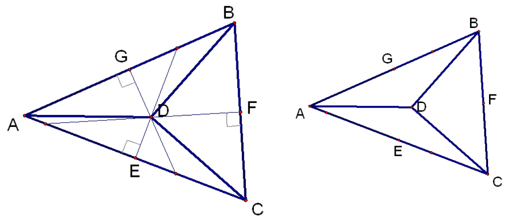
Circumcenter—the point of concurrency of the perpendicular bisectors of a triangle.



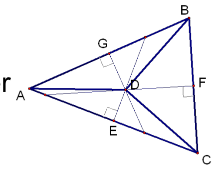


Thm 5.4 Circumcenter theorem—The circumcenter of a triangle is equidistant from the vertices of the triangle.

Conclusion: $\overline{AD} \cong \overline{CD} \cong \overline{BD}$

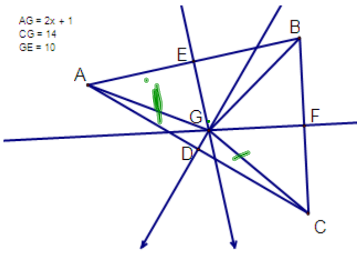


Proof:
Given: D is the circumcenter
Prove: $\overline{AD}$ $\overline{CD}$ $\overline{BD}$



Ex 2. G is the circumcenter. x =

AG = 2x + 1
CG = 14
GE = 10



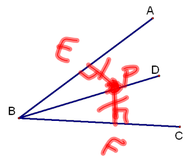
AG = CG
$2x+1=14$
$2x=13$
$x=6.5$

5.3 Angle Bisectors

Thm 5.5—Angle Bisector Theorem--Any point on the angle bisector is equidistant from the sides of the angle.

Thm 5.6—Converse of the Angle Bisector Theorem—If a point is on the interior of an angle and it is equidistant from the sides of the angle, then it lies on the angle bisector.

Conclusion: $\overline{PE} \cong \overline{PF}$



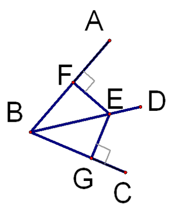
Ex 3. $\overleftrightarrow{BD}$ bisects ∠ABC. Solve for x.________

m∠ABC = 15x

m∠ABD = 10x – 8

$2(10x-8)=15x$

$x=\frac{16}{5}$



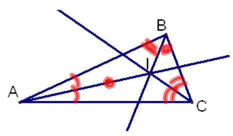
EF = y + 8

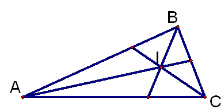
EG = 3y – 6

y = 7

$y+8=3y-6$

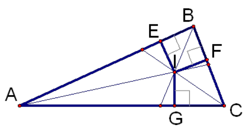
Incenter—The point of concurrency of the angle bisectors of a triangle.

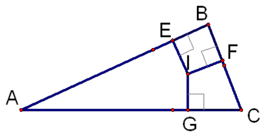




Thm 5.7 The Incenter Theorem—The incenter of a triangle is equidistant from each side of the triangle.

Conclusion: $\overline{IG} \cong \overline{IF} \cong \overline{EI}$

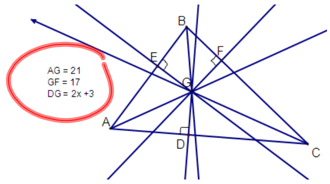




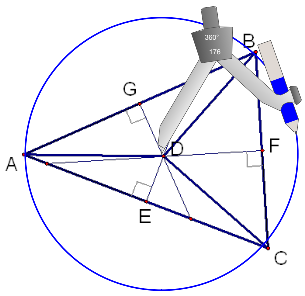
Ex 4. G is the incenter. x = 7.

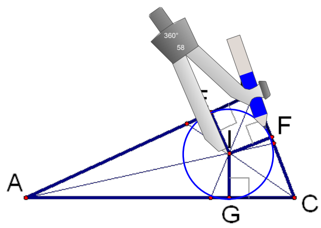
$17 = 2x + 3$

$7 = x$



Circles can be drawn around these centers.





HW
p307 #s 11-17
p309 quiz #s 1-3
p313-314 #s 3-8, 12-17